



|                                 |                              |
|---------------------------------|------------------------------|
| Name Surname: <b>ANSWER KEY</b> | Group No:                    |
| Student No:                     | Duration: 90 mins.           |
| Department:                     | Date: Saturday, Nov 27, 2021 |
| Lecturer:                       | Signature:                   |

**! Attention:** The 9th article of Student Disciplinary Regulations of Higher Education Council (YÖK) Law No. 2547 states that people who are "cheating or helping to cheat or attempt to cheat in exams" will be punished by suspension of one or two semesters.

1. Let  $A$  be a  $4 \times 4$  matrix with real entries and assume that  $A^{-1}$  exists. Which of the following is always true?

- a)  $\text{Rank}(A) = 1$     b)  $\det(A) = 1$     c)  $\det(A) = 4$   
d)  $\text{Rank}(A) = 4$     e)  $\det(A) = \text{Rank}(A)$

2. For which value of  $k$ , the linear system corresponding to the augmented matrix

$$\left[ \begin{array}{cccc|c} 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & -1 & 1 & -1 \\ 4 & 5 & 0 & k & 2 \end{array} \right]$$

is inconsistent?

- a) 2    b) 4    c) 6    d)  $k \in \mathbb{R}$   
e) There is no such  $k$

3. Let  $A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 0 & -2 \\ 1 & 3 & -2 \end{bmatrix}$  be a nonsingular matrix. Which of the following matrices is equal to  $\text{Adj}(A^{-1})$ ?

- a)  $\begin{bmatrix} 3/7 & 11/14 & -1/7 \\ 1/7 & -1/14 & 2/7 \\ 3/7 & 2/7 & -1/7 \end{bmatrix}$     b)  $\begin{bmatrix} -1/14 & 1/14 & 3/14 \\ 1/7 & 0 & -1/7 \\ 1/14 & 3/14 & -1/7 \end{bmatrix}$   
c)  $\begin{bmatrix} 3/7 & 1/7 & 3/7 \\ 11/14 & -1/14 & 2/7 \\ -1/7 & 2/7 & -1/7 \end{bmatrix}$     d)  $\begin{bmatrix} -1/14 & 1/7 & 1/14 \\ 1/14 & 0 & 3/14 \\ 3/14 & -1/7 & -1/7 \end{bmatrix}$   
e) None of them

4. Which of the following statement(s) are true?

- I. Every matrix in row echelon form is also in reduced row echelon form  
II. The reduced row echelon form of a nonsingular matrix is an identity matrix  
III. The reduced row echelon form of a singular matrix has a row of zeros  
IV. Any matrix equivalent to an identity matrix is singular  
a) I and IV    b) Only I    c) II and III    d) Only IV  
e) All of them

5. Compute the determinant of the following matrix.

$$\begin{bmatrix} 2 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & 7 \\ -3 & 2 & 0 & -1 & -6 \\ 2 & -2 & -1 & 1 & 4 \\ 0 & 0 & 0 & 4 & 3 \end{bmatrix}$$

- a) 0    b) -56    c) 56    d) -112    **e) 112**

6. For which choice(s) of the constant  $k$  is the following matrix invertible?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$$

- a)  $k = 1$  or  $k = 2$     b)  $\mathbb{R} - \{1\}$     **c)  $\mathbb{R} - \{1, 2\}$**   
 d) For all  $k \in \mathbb{R}$     e) There do not exist such a constant  $k$ .

7. Let  $A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$  be an involutory matrix,  $B = \begin{pmatrix} y & 0 \\ y & 0 \end{pmatrix}$  a nonzero idempotent matrix and  $C = \begin{pmatrix} 1 & -3 \\ z & 3 \end{pmatrix}$  a singular matrix. What is the value of  $x - y - z$ ?

- a) -1    **b) 0**    c) 1    d) -2    e) 2

8. Consider the matrix  $A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{bmatrix}$ . Which of the following is the inverse of  $A$ , (if exists)? (Hint: You may use elementary row operations.)

a)  $\begin{bmatrix} -4/5 & 3/2 & 1/2 & 4/5 \\ 3/5 & 0 & 0 & 2/5 \\ 1/5 & -1 & 0 & -1/5 \\ 1/5 & 0 & 0 & -1/5 \end{bmatrix}$

**b)  $\begin{bmatrix} -4/5 & 3/5 & 1/5 & 1/5 \\ 3/2 & 0 & -1 & 0 \\ 1/2 & 0 & 0 & 0 \\ 4/5 & 2/5 & -1/5 & -1/5 \end{bmatrix}$**

c)  $\begin{bmatrix} -4/5 & 3/5 & 3/2 & 4/5 \\ 1/5 & 0 & 0 & 1/5 \\ 2/5 & -1 & 0 & -2/5 \\ 2/5 & 0 & 0 & -2/5 \end{bmatrix}$

d)  $\begin{bmatrix} -4 & 3 & 1 & 4 \\ 3/5 & 0 & 0 & 2/5 \\ 1 & -1 & 0 & -1 \\ 5 & 0 & 0 & -1 \end{bmatrix}$     e)  $A$  is noninvertible.

9. If  $A$  and  $B$  are  $3 \times 3$  matrices with real entries such that  $\det(A) = \sqrt{3}$  and  $\det(B) = \frac{1}{2}$ , what is the value of  $\det(2A^T B^{-3})$ ?

- a)  $32\sqrt{3}$     b)  $16\sqrt{3}$     c) 48    d)  $4\sqrt{3}$     **e)  $64\sqrt{3}$**

10. Find the solution(s) to the homogeneous system

$$\begin{aligned} 4x_1 - 2x_2 + 7x_3 &= 0 \\ 8x_1 - 3x_2 + 10x_3 &= 0 \end{aligned}$$

a) It has only a trivial (zero) solution.

b)  $x_1 = \frac{1}{4}k, x_2 = 4k, x_3 = k, k \in \mathbb{R}.$

c)  $x_1 = 4k, x_2 = \frac{1}{4}k, x_3 = k, k \in \mathbb{R}.$

d)  $x_1 = k, x_2 = 4k, x_3 = k, k \in \mathbb{R}.$

e) The system is inconsistent (no solution).

11. Assume that  $\text{rank}(A) = 2$  for the matrix

$$A = \begin{bmatrix} a & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & 1-a \end{bmatrix}. \text{ Find the value of } a.$$

a)  $a = -1$  or  $a = -2$

b)  $a = -2$  or  $a = 2$

c)  $a = -1$  or  $a = 1$

d)  $a \neq -1$  or  $a \neq -2$

e)  $a = -1$  or  $a = 2$

12. Which of the following statement(s) for determinants concerning  $n \times n$  matrices  $A$  and  $B$  are correct?

I.  $\det(A^T B) = \det(B^T A).$

II. If the determinant of a  $3 \times 3$  ( $n = 3$ ) matrix is equal to the product of its elements in the main diagonal then it is either an upper triangular or a lower triangular matrix.

III. For a  $4 \times 4$  ( $n = 4$ ) matrix  $B$  one always has  $\det(2B) = 16 \det(B).$

a) Only I

b) I and II

c) II and III

d) I and III

e) All of them

13. Let  $A = \begin{bmatrix} 0 & -3 & -1 & 1 \\ -2 & 0 & 2 & 5 \\ 3 & -2 & 0 & 0 \\ 1 & -4 & 0 & 0 \end{bmatrix}.$  Which of the following statement(s) are correct for the matrix  $A$  ?

I.  $A$  has an inverse.

II. The reduced row echelon form of  $A$  is a  $4 \times 4$  identity matrix.

III.  $\text{Adj}(A)A = \begin{bmatrix} 70 & 0 & 0 & 0 \\ 0 & 70 & 0 & 0 \\ 0 & 0 & 70 & 0 \\ 0 & 0 & 0 & 70 \end{bmatrix}$

a) Only I

b) I and II

c) I and III

d) II and III

e) I, II and III

14. Let  $A$  be a  $3 \times 3$  matrix such that the sum of the columns of  $A$  is equal to  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$  Let  $B = \begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}.$  Which of the following could be the product matrix  $AB$ ?

a) None of them

b)  $\begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

c)  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e)  $\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

15. Let  $A$  be a  $3 \times 3$  matrix with  $\det(A) = -7.$  Which of the following is the value of  $\det(2A^{-1} + \text{adj}(A))$  ?

a)  $\frac{125}{7}$

b)  $-125$

c)  $\frac{-125}{7}$

d)  $\frac{-1}{7}$

e)  $\frac{-7}{125}$

16. For what value(s) of  $a$  does the following linear system have a unique solution?

$$\begin{aligned}x + y - z &= 2 \\x + 2y + z &= 3 \\x + y + (a^2 - 5)z &= a\end{aligned}$$

- a)  $a = 2$     b)  $a = -2$     c)  $a = \pm 2$      d)  $a \neq \pm 2$   
e) There is no such  $a$ .

18. If  $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4$ , which of the following is equal to  $\begin{vmatrix} a_1 & a_2 & 4a_3 - 2a_2 \\ b_1 & b_2 & 4b_3 - 4b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 2c_3 - c_2 \end{vmatrix}$  ?
- a) 8    b) 6    c) 4    d) 2    e) 1

19. If  $A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$  and  $AB = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}$ , which of the following is the  $(2, 4)$ -entry ( $b_{24}$ ) of  $B$  ?
- a)  $-2$     b)  $3$     c)  $8$      d)  $-4$     e)  $11$

17. Let  $A$  be a nonsingular matrix whose inverse is  $A^{-1} = \begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}$  and let  $b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ . Which of the following is a solution of the linear system  $A^T x = b$  ?

- a)  $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$     b)  $x = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$      c)  $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$   
d)  $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$     e)  $x = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

20. Determine the value of  $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$  for the matrix  $A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix}$ , where  $A_{ij}$  is the cofactor of each  $a_{ij}$  ?
- a) 0    b)  $-32$     c) 32    d) 16    e)  $-16$